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FURTHER ANALYSIS OF GAMMA RAY ATTENUATION IN TWO-LEGGED RECTANGULAR DUCTS

Sequel To TN-383

TN-412

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U. S. NAVAL CIVIL ENGINEERING LABORATORY

Port Hueneme, California

FURTHER ANALYSIS OF GAMMA RAY ATTENUATION IN TWO-LEGGED RECTANGULAR DUCTS

Sequel To TN-383

Y-F011-05-329

Type C

by

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OBJECT OF TASK

To improve existing knowledge of gamma and neutron shielding properties of shelters.

ABSTRACT

The use of the albedo approach for the analytical prediction of attenuation characteristics of two-legged rectangular ducts transmitting gamma rays is extended to cases involving the asymmetrical location of source and/or detector. Illustrative problems are presented and solved.

I. INTRODUCTION

The problem of radiation streaming down ducts and passageways in a shield has many facets. The part of the problem relating to attenuation of gamma radiation through two-legged rectangular ducts, with symmetrical placement of source and detector, has been the subject of several recent investigations, both experimental and analytical (References 1-3).

This note carries on the program of analysis initiated in Reference 1 and undertakes an analytical solution of similar problems for cases where the source and/or the detector are not located on the axis of symmetry of the legs, but are to one side.

II. BACKGROUND

Reference 1 has displayed in some detail a method of predicting the attenuation of gamma radiation as it passes down a two-legged rectangular duct within a thick shield. There is therefore no need here for a precise exposition of the general problem and the precise assumptions that are necessary to obtain analytical solutions. It is assumed that the reader has become acquainted with such matters from Reference 1.

An outline of the problem, however, is desirable. Its main aspects are as follows:

(a) We are concerned with ducts of rectangular cross section, having more than one leg, with the legs intersecting perpendicularly. These ducts may be as small as an air shaft, or they may be as large as an entranceway to a personnel shelter designed to provide protection against atomic bomb radiation.

(b) The minimum dimension of the ducts is several inches, and the lengths are less than, say, a hundred feet. The ratio of length to width or height is considered to be three or more. (Reference 1, however, indicates surprisingly accurate results at ratios almost down to unity.)

(c) A high order of accuracy is sought by eliminating any known approximations giving errors greater than 10 percent; however, an overall ratio between analysis and experiment of not over 1.5 is considered adequate.

(d) The analytical approach employs the "albedo" concept. Multiple scattering off walls is ignored, for lack of adequate means of handling this aspect of the problem at present.

(e) The wall material is uniform in composition and density.

III. THE PROBLEM OF OFF-CENTERED SOURCE AND/OR DETECTOR

In Reference 1 the general problem was simplified by assuming all the source radiation to be concentrated at the center of the entrance to the first duct leg (or assuming by symmetry that it could be so regarded), and by having the detector at the center of the exit of the second duct leg. There are circumstances, however, where this assumption should not or cannot be made. As a further refinement, therefore, one should investigate the cases in which source and/or detector are placed at the "farther" or "nearer" wall of the duct system. Figures 1, 2, and 3 illustrate the various situations studied.

It can be readily demonstrated that the position of the source away from the center, in the direction perpendicular to the plane established by the axes of the duct legs, does not significantly affect the analytical results; whereas change of location in the plane of the axis does. We therefore need not concern ourselves about location at the other two sides of the access openings (the floor and ceiling of a horizontal entranceway, for example). The "far" or "near" wall cases are to be considered only.

The procedure in each case is almost precisely the same as that given in Reference 1, and we shall not include all detailed derivations in this treatise. Only the pertinent results will be noted.

Table 1 indicates how the contributions to the detector response in various cases are readily established by comparison with that of the fundamental case analyzed in Reference 1, the so-called C-C case. By using this table, one can obtain and add the various wall scattering contributions to the overall detector readings, for the various cases. We can, then, for each case use the same fundamental formulas provided in Reference 1 for the C-C case:

$$F_T = F_1 F_2$$

$$F_2 = (4\beta_1 \beta_2 \beta_3)\, G_{TOT}$$

$$G_{TOT} = G_b + G_{t1} + G_{t2} + G_s$$

in which F_T is the overall attenuation factor D/D_o, F_1 is the attenuation factor for the first leg (not particularly sensitive to location of source across the entranceway), F_2 is the attenuation factor for the second leg (of prime concern to us here), and the G's are terms or combinations of terms (largely geometrical in nature) which permit ready calculation of the attenuation factors.

The various G-terms for the several cases are listed below. For G_b the following are readily derived.

Case C-C:

$$G_b = \frac{a_1}{1+\beta_1} + \frac{a_2}{\beta_2(1+\beta_2)} + \frac{a_3}{1-\beta_1} + \frac{a_4}{1-\beta_2}$$

Case F-C:

$$G_b = \frac{a_1}{1+\beta_1} + \frac{a_2}{\beta_2} + \frac{a_3}{1-\beta_1} + \frac{a_4}{(1-\beta_2)^2}$$

Case N-C:

$$G_b = \frac{a_1}{1+\beta_1} + \frac{a_2}{\beta_2(1+\beta_2)^2} + \frac{a_3}{1-\beta_1} + a_4$$

Case C-F:

$$G_b = a_1 + \frac{a_2}{\beta_2(1+\beta_2)} + \frac{a_3}{(1-\beta_1)^2} + \frac{a_4}{(1-\beta_2)}$$

Case F-F:

$$G_b = a_1 + \frac{a_2}{\beta_2} + \frac{a_3}{(1 - \beta_1)^2} + \frac{a_4}{(1 - \beta_2)^2}$$

Case N-F:

$$G_b = a_1 + \frac{a_2}{\beta_2 (1 + \beta_2)^2} + \frac{a_3}{(1 - \beta_1)^2} + a_4$$

Case C-N:

$$G_b = \frac{a_1}{(1 + \beta_1)^2} + \frac{a_2}{\beta_2 (1 + \beta_2)} + a_3 + \frac{a_4}{(1 - \beta_2)}$$

Case F-N:

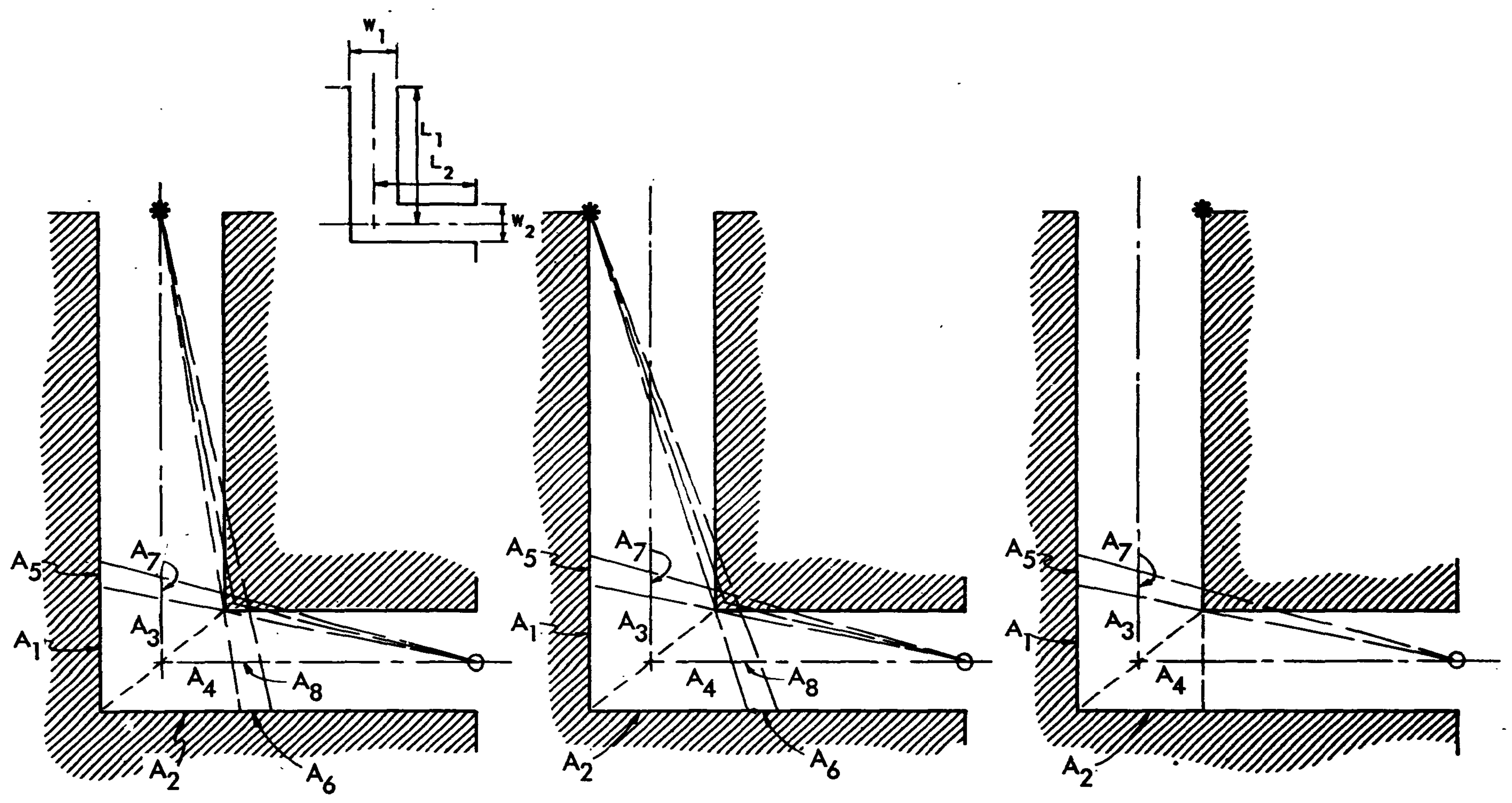
$$G_b = \frac{a_1}{(1 + \beta_1)^2} + \frac{a_2}{\beta_2} + a_3 + \frac{a_4}{(1 - \beta_2)^2}$$

Case N-N:

$$G_b = \frac{a_1}{(1 + \beta_1)^2} + \frac{a_2}{\beta_2 (1 + \beta_2)^2} + a_3 + a_4$$

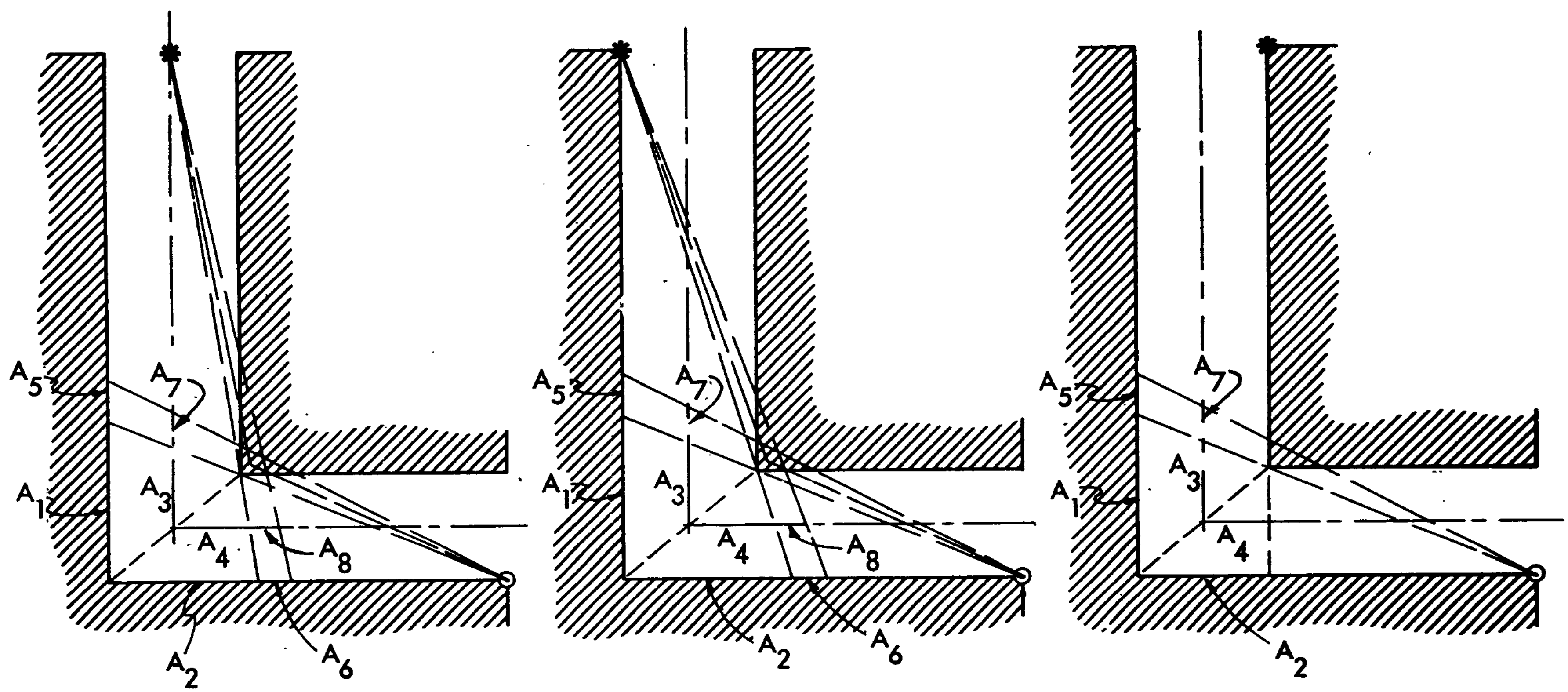
G_{t1} is the same for cases C-C, F-C, and N-C; it is as given in Reference 1. For cases C-F, F-F, and N-F, it is twice as much as for the fundamental C-C case. For cases C-N, F-N, and N-N, it is zero.

G_{t2} is the same for cases C-C, C-F, and C-N; it is as given in Reference 1. For cases F-C, F-F, and F-N, it is twice as much as for the fundamental C-C case. For cases N-C, N-F, and N-N, it is zero.



"C-C" Case "F-C" Case "N-C" Case

Figure 1. Duct geometry for cases with detector centered.

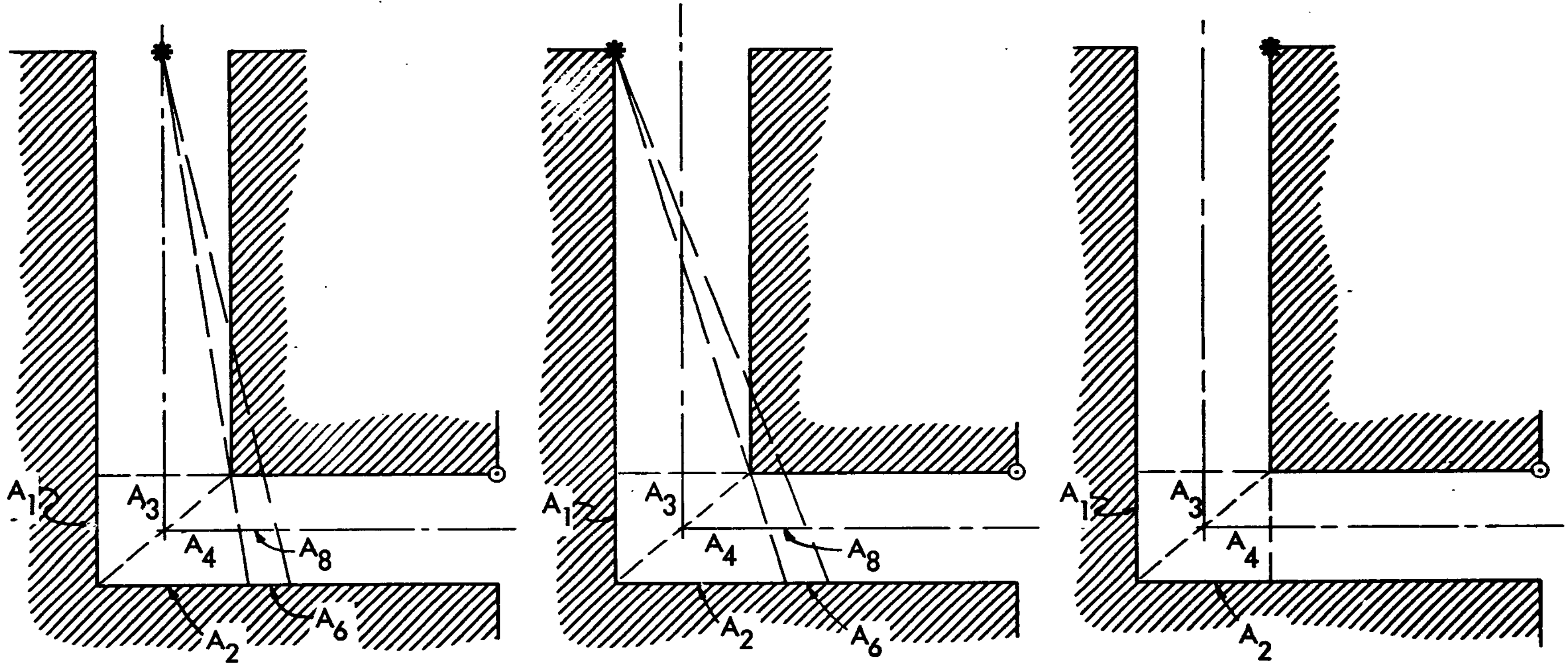


"C-F" Case

"F-F" Case

"N-F" Case

Figure 2. Duct geometry for cases with detector at the farther side.



"C-N" Case

"F-N" Case

"N-N" Case

Figure 3. Duct geometry for cases with detector at the nearer side.

Table I. Relative Values of Scattering Areas and Associated Dose Contributions

Area \ Case	C–C	F–C	N–C	C–F	F–F	N–F	C–N	F–N	N–N
1	1	1	1	(a)	(a)	(a)	(c)	(c)	(c)
2	1	(b)	(d)	1	(b)	(d)	1	(b)	(d)
3	1	1	1	(a)	(a)	(a)	(c)	(c)	(c)
4	1	(b)	(d)	1	(b)	(d)	1	(b)	(d)
5	1	1	1	2	2	2	0	0	0
6	1	2	0	1	2	0	1	2	0
7	1	1	1	2	2	2	0	0	0
8	1	2	0	1	2	0	1	2	0

Key to cases: C indicates location in Center.
F indicates location at Farther Wall.
N indicates location at Nearer Wall.
The first letter denotes location of source; the second letter denotes location of detector. For example, C–F means that the source is located at the center of the entrance to the first leg, and the detector is located on the farther wall at the end of the second leg.

Key to tabulated entries: Values are relative to the C–C case, which is given in Tables I and III of Reference 1.
(a) indicates $1 + \beta_1$, or $1/(1 - \beta_1)$.
(b) indicates $1 + \beta_2$, or $1/(1 - \beta_2)$.
(c) indicates $1 - \beta_1$, or $1/(1 + \beta_1)$.
(d) indicates $1 - \beta_2$, or $1/(1 + \beta_2)$.

Note: $\beta_1 = \frac{W_1}{2 L_2}$, $\beta_2 = \frac{W_2}{2 L_1}$, $\beta_3 = \frac{H}{2 L_2}$.

The various expressions for G_s are readily obtained by carrying through, for each case, the same sort of derivation given in Reference 1 for the C-C case. In general, the following expressions are obtained.

$$\alpha_1 = \tan^{-1} \frac{K_1 W_1}{2 L_1 (1 - \beta_2)}$$

$$\alpha_2 = \tan^{-1} \frac{K_2 W_2}{2 L_2 (1 - \beta_1)}$$

$$\theta_s = 90^\circ - \alpha_1 - \alpha_2$$

$$G_s = \frac{K_3 Z N K(\theta_s, E_o)}{2 \mu_a^2 L_2 (1 - \beta_1)^3 (1 - \beta_2)^3}$$

The values of K_1, K_2, and K_3 are given in Table II for the various cases.

Table II. The Values of the Constants K in Equations

Case	K_1	K_2	K_3
C-C	1	1	1
F-C	2	1	2
N-C	-	-	0
C-F	1	2	2
F-F	2	2	4
N-F	-	-	0
C-N	-	-	0
F-N	-	-	0
N-N	-	-	0

IV. EXAMPLE NO. 1

Problem. Consider a two-legged duct, of square cross section, 28.2 centimeters on a side. The walls are of lightweight concrete, 124 pcf. Each leg is 80 centimeters long. A point source, with a photon energy assumed to be 0.34 Mev, is located at the "far" side of the entrance to the duct. The detector is located on the center line at the exit. What is the overall attenuation of the duct, using the response of a detector one centimeter away from the source in free space as a reference?

Solution. The solution is carried out as indicated in Table III herein, which has essentially the same format as prescribed for this kind of problem by Table V in Reference 1. The case herein is the F-C case and the formulas must be adjusted as described in Section III above. The attenuation is 1.72×10^{-6}.

V. EXAMPLE NO. 2

Problem. Figure 4 illustrates a duct system with a parallel, broad beam of 0.5-Mev gamma photons incident at an angle $\psi = 38^{\circ}$ with the normal to the far side of the first duct leg. The duct dimensions and materials are the same as in the previous example, except that the length of the first leg is 91 centimeters. If the outside dose, within the beam, is 100,000 roentgens, what dose is registered by the detector at the end of the second leg? (Assume no direct penetration of the shield itself.)

Discussion. Let us briefly review the albedo concept before proceeding with the solution. (Figure 5, which is the same as Figure 1 of Reference 1, is reproduced for reference purposes.)

The basic albedo equation, Equation 1 of Reference 1, can be written as follows:

$$D = (D_o)\left(\frac{1}{r_1^{\,2}}\right)(A\,\bar{a}\cos\theta_1)\left(\frac{1}{r_2^{\,2}}\right)$$

Let us consider the factors on the right-hand side of the equation, from left to right. The first term gives the reading of the detector at the original reference point. Multiplied by the next factor, it provides a detector response at the scattering area without scattering, and thus indicates the level of radiation striking the surface. Multiplied then by the third factor, it gives the detector reading a unit distance away from the scattering surface after and including only the scattering from the area A. (This latter detector reading is somewhat hypothetical unless the dimensions of the scattering surface are much less than unity — it is nevertheless a useful "mental crutch.") When the final factor is also included, we get the detector response at the distance r_2 from the scattering surface. Thus the scattering surface, if its dimensions are small compared to r_2, acts essentially as a point source for radiation going along path r_2.

Under conditions where the incident radiation does not proceed from a point source but is a broad, parallel beam incident at angle ψ to the normal, it is readily seen that the first two factors are replaced simply by the detector reading in the unscattered beam, which we will call D_u. Thus the albedo equation would be:

$$D = D_u A \bar{a} \cos\psi \left(\frac{1}{r_2^{\,2}}\right)$$

In such case, we see, the scattering area A acts as if it were a point source with sufficient strength for the detector reading one unit distance from it to give a value of $D_u A \bar{a} \cos\psi$.

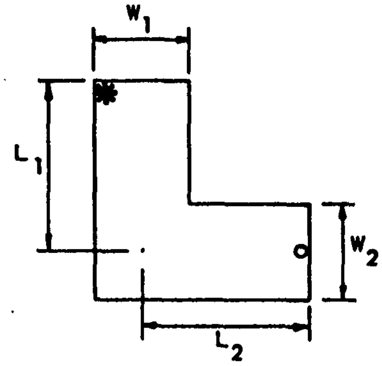
Solution. The problem quickly reduces to the sort of two-legged duct problem we have discussed in this paper, the F-C case, if we take the mid-point of the scattering area at the entrance side wall as the location of the source in the reduced problem. The strength of the source in the reduced problem, which we will call D_o', is, as explained in the above discussion:

$$D_o' = D_u A \bar{a} \cos\psi$$

$$= D_u (W_1 H \tan\psi) \bar{a} \cos\psi$$

$$= D_u W_1 H \bar{a} \sin\psi$$

Table III. Solution to Problem No. 1 (page one of two)



$L_1 = 80$ cm

$L_2 = 80$ cm

$W_1 = 28.2$ cm

$W_2 = 28.2$ cm

$H = 28.2$ cm

$$\beta_1 = \frac{W_1}{2 L_2} \qquad \beta_2 = \frac{W_2}{2 L_1} \qquad \beta_3 = \frac{H}{2 L_2}$$

	$1 + \beta$	$1 - \beta$	$(1 - \beta)^2$	$(1 - \beta)^3$
β_1	1.176	0.824	0.679	0.559
β_2	1.176	0.824	0.679	0.559

Source Energy, E_o = 0.34 Mev

Scatt. Area	$\cos\theta_1$	
	Formula	Value
1	$W_1/2L_1$	0.176
2	1.00	1.00
3	$H/2L_1$	0.176
4	$H/2L_1$	0.176
5	$\frac{W_1}{2 L_1 (1 - \beta_2)}$	0.214
6	1.00	1.00
7	$\frac{W_1}{2 L_1 (1 - \beta_2)}$	0.214
8	$H/2L_1$	0.176

See References 1 and 4 to obtain values of a_i, using arguments of E_o and $\cos\theta_1$

$\beta_1 = 0.176$

$\beta_2 = 0.176$

$\beta_3 = 0.176$

$\mu_a = 0.0594$

$\mu_a' = 0.0594$

$$\left(\frac{\frac{ZN}{2}}{\mu_a}\right) r_o^2 = 13.5$$

$\alpha_1 = 23.2°$

$\alpha_2 = 12.1°$

$\theta_s = 54.7°$

$K/r_o^2 = 0.28$

$a_1 = 0.046$

$a_2 = 0.015$

$a_3 = 0.046$

$a_4 = 0.046$

$a_5 = 0.043$

$a_6 = 0.015$

$a_7 = 0.043$

$a_8 = 0.046$

Table III. Solution to Problem No. 1, continued (page two of two)

For the F-C case:

$$G_b = \frac{a_1}{1+\beta_1} + \frac{a_2}{\beta_2} + \frac{a_3}{1-\beta_1} + \frac{a_4}{(1-\beta_2)^2}$$

$$= 0.039 + 0.085 + 0.056 + 0.068 \qquad G_b = 0.248$$

$$G_t = \frac{(1-\beta_1)\ a_5 - 2\,a_7}{2\mu_a'\, L_2\,(1-\beta_1)^2\ (1-\beta_2)^3}$$

$$+ \frac{(1-\beta_2)\ a_6 - 2\,\beta_2\,a_8}{\mu_a\, L_1\, \beta_2\,(1-\beta_1)^2\ (1-\beta_2)^2}$$

$$= \frac{0.1214}{3.60} + \frac{0.0286}{0.386}$$

$$= 0.0337 + 0.0740 \qquad G_t = 0.108$$

$$G_s = \left(\frac{Z\,N\,K}{\mu_a^{\,2}}\right)\left(\frac{1}{L_2\,(1-\beta_1)^3\ (1-\beta_2)^3}\right)$$

$$= 3.78/25.0 \qquad G_s = 0.151$$

$$G_{TOT} = 0.507$$

$F_1 = 1/L_1^{\,2}$	$F_1 = 1.10 \times 10^{-2}$
$F_2 = 4\beta_1\beta_2\beta_3\,G_{TOT}$	$F_2 = 1.56 \times 10^{-4}$
$F_T = F_1\,F_2$	$F_T = 1.72 \times 10^{-6}$

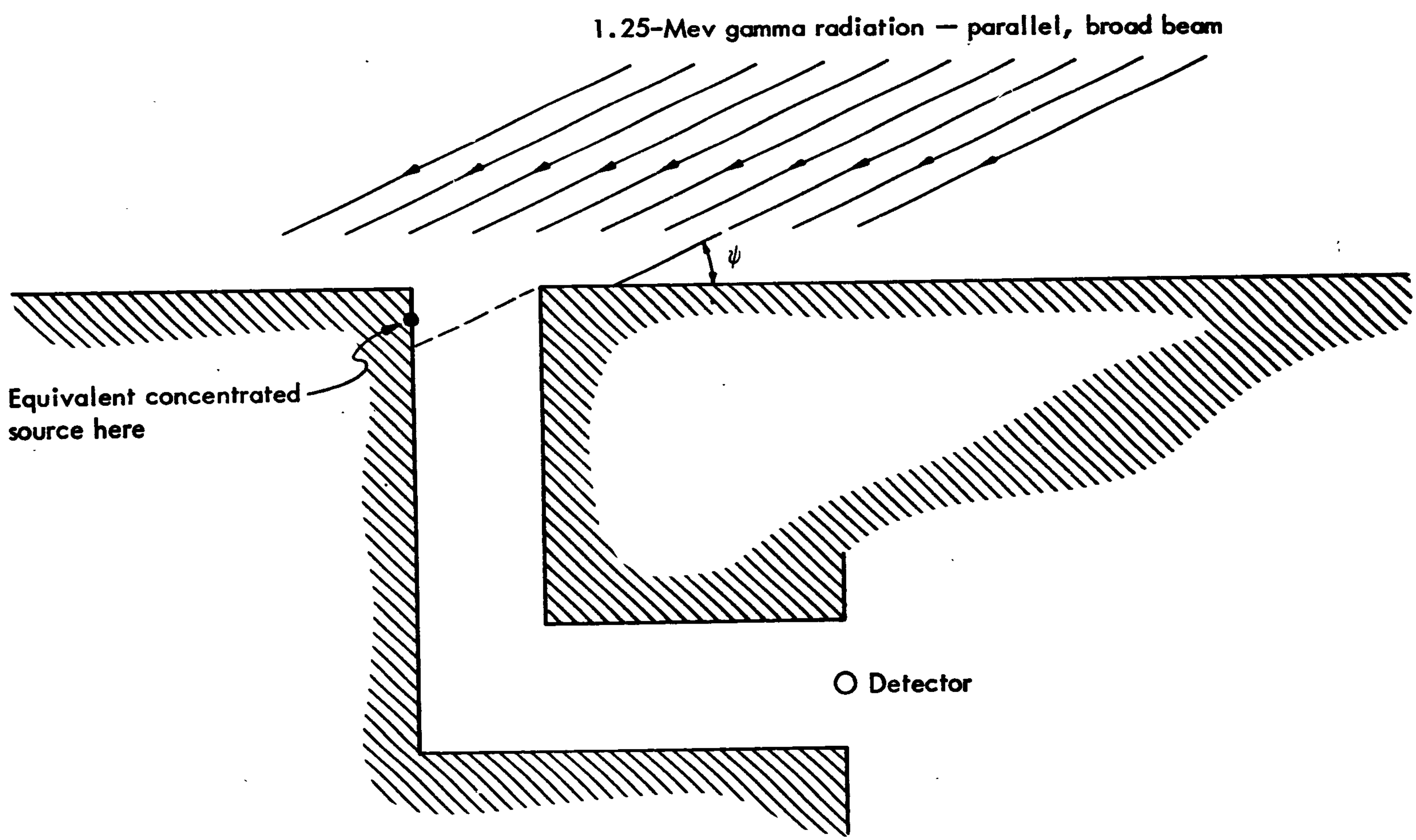


Figure 4. Illustration for Example No. 2.

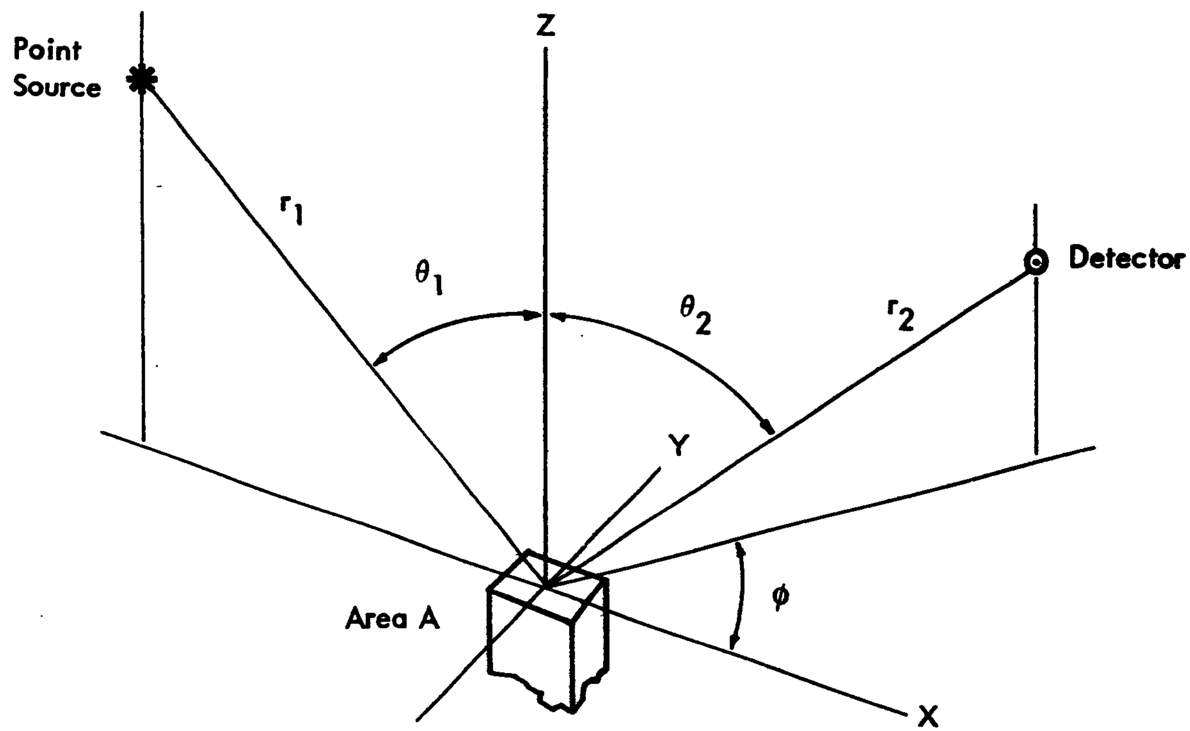


Figure 5. Scattering of gamma rays from surface.

Substituting the given numerical data and obtaining the value of $\bar{a}$ from Figure 4, Reference 1, we get:

$$D_o' = (100{,}000)\,(28.2)\,(28.2)\,(0.616)\,(0.0148)$$

$$= 725{,}000 \text{ roentgens-cm}^2.$$

(The fact that this figure is, astonishingly, larger than the value of D_u, the reading in open air, is due to the hypothetical nature of the concept that the radiation scattered from an area of the side wall near the opening is considered to be concentrated into a single point.)

The radiation scattered off the far wall at the entrance may be considered, without undue error, as coming from a single point on the wall, at a distance from the opening equal to $1/2\, W_1 \tan\psi$, which here has a numerical value of 11 centimeters. For the reduced problem, then, the length of the first leg is 91 minus 11, or 80 centimeters.

The next quantity needed is the representative energy for the gamma photons after scattering. Assuming that most of the albedo is due to single scattering, we can use the usual Compton scattering relationship (Reference 5, p. 146) to determine the scattered photon energy. From geometrical relationships, the average scattering angle is found to be 62 degrees; by use of the Compton relationship, the scattered photon energy is found to be about 0.34 Mev.

At this point we solve the reduced problem for a two-legged duct attenuation, using the F-C case. For this particular example, the solution has already been obtained in Example 1, above. The dose recorded by the detector at the end of the second leg of the duct is therefore found to be $(1.72 \times 10^{-6}) \times (0.725 \times 10^{6})$, or 1.25 roentgens.

VI. CONCLUDING REMARKS

We have no experimental data to confirm adequately the type of analysis derived and utilized herein. The results of this work therefore cannot be said to have achieved firm scientific status. It is, however, based on a plausible extension of principles and techniques which are more firmly established, and for the time being will serve to permit engineering calculations for these and related practical situations. The need for confirming experimental work is clear.

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